Package 'moocore'

September 18, 2024

Type Package

Title Core Mathematical Functions for Multi-Objective Optimization

Version 0.1.2

Description
Fast implementation of mathematical operations and performance metrics for multi-objective optimization, including filtering and ranking of dominated vectors according to Pareto optimality, computation of the empirical attainment function, V.G. da Fonseca, C.M. Fonseca, A.O. Hall (2001) <doi:10.1007/3-540-44719-9_15>, hypervolume metric, C.M. Fonseca, L. Paquete, M. López-Ibáñez (2006) <doi:10.1109/CEC.2006.1688440>, epsilon indicator, inverted generational distance, and Vorob'ev threshold, expectation and deviation, M. Binois, D. Ginsbourger, O. Roustant (2015) <doi:10.1016/j.ejor.2014.07.032>, among others.

Depends R (>= 4.0)

Imports matrixStats, Rdpack

Suggests doctest (>= 0.2.0), knitr, spelling, testthat (>= 3.0.0), withr

License LGPL (>= 2)

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BugReports https://github.com/multi-objective/moocore/issues

URL https://multi-objective.github.io/moocore/r/, https://github.com/multi-objective/moocore/tree/main/r

LazyLoad true

LazyData true

Encoding UTF-8

RoxygenNote 7.3.2

SystemRequirements GNU make

RdMacros Rdpack

Config/testthat/edition 3

Language en-GB

Config/Needs/website rmarkdown, reshape2, ggplot2

NeedsCompilation yes

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Repository CRAN

Date/Publication 2024-09-18 04:50:02 UTC

Contents

`as_double_matrix` *Convert input to a matrix with* `"double"` *storage mode (*`base::storage.mode()`*).*

Description

Convert input to a matrix with `"double"` storage mode (`base::storage.mode()`).

Usage

```
as_double_matrix(x)
```

Arguments

`x`	`data.frame()|matrix()` A numerical data frame or matrix with at least 1 row and 2 columns.

Value

`x` is coerced to a numerical `matrix()`.

`attsurf2df` *Convert a list of attainment surfaces to a single EAF* `data.frame`.

Description

Convert a list of attainment surfaces to a single EAF `data.frame`.

Usage

```
attsurf2df(x)
```

Arguments

`x`	`list()` List of `data.frames` or matrices. The names of the list give the percentiles of the attainment surfaces. This is the format returned by `eaf_as_list()`.

Value

`data.frame()`
Data frame with as many columns as objectives and an additional column `percentiles`.

See Also

`eaf_as_list()`

Examples

```
data(SPEA2relativeRichmond)
attsurfs <- eaf_as_list(eaf(SPEA2relativeRichmond, percentiles = c(0,50,100)))
str(attsurfs)
eaf_df <- attsurf2df(attsurfs)
str(eaf_df)
```

`choose_eafdiff` *Interactively choose according to empirical attainment function differences*

Description

Interactively choose according to empirical attainment function differences

Usage

```
choose_eafdiff(x, left = stop("'left' must be either TRUE or FALSE"))
```

Arguments

`x`	`matrix()` Matrix of rectangles representing EAF differences returned by `eafdiff()` with `rectangles=TRUE`.
`left`	`logical(1)` With `left=TRUE` return the rectangles with positive differences, otherwise return those with negative differences but differences are converted to positive.

Value

`matrix()` where the first 4 columns give the coordinates of two corners of each rectangle and the last column. In both cases, the last column gives the positive differences in favor of the chosen side.

Examples

```
extdata_dir <- system.file(package="moocore", "extdata")
A1 <- read_datasets(file.path(extdata_dir, "wrots_l100w10_dat"))
A2 <- read_datasets(file.path(extdata_dir, "wrots_l10w100_dat"))
# Choose A1
rectangles <- eafdiff(A1, A2, intervals = 5, rectangles = TRUE)
rectangles <- choose_eafdiff(rectangles, left = TRUE)
reference <- c(max(A1[, 1], A2[, 1]), max(A1[, 2], A2[, 2]))
x <- split.data.frame(A1[,1:2], A1[,3])
hv_A1 <- sapply(split.data.frame(A1[, 1:2], A1[, 3]),
                hypervolume, reference=reference)
hv_A2 <- sapply(split.data.frame(A2[, 1:2], A2[, 3]),
                hypervolume, reference=reference)
print(fivenum(hv_A1))
```

```
print(fivenum(hv_A2))
whv_A1 <- sapply(split.data.frame(A1[, 1:2], A1[, 3]),
                 whv_rect, rectangles=rectangles, reference=reference)
whv_A2 <- sapply(split.data.frame(A2[, 1:2], A2[, 3]),
                 whv_rect, rectangles=rectangles, reference=reference)
print(fivenum(whv_A1))
print(fivenum(whv_A2))
```

`compute_eafdiff_call` *Same as* `eafdiff()` *but performs no checks and does not transform the input or the output. This function should be used by other packages that want to avoid redundant checks and transformations.*

Description

Same as `eafdiff()` but performs no checks and does not transform the input or the output. This function should be used by other packages that want to avoid redundant checks and transformations.

Usage

```
compute_eafdiff_call(x, y, cumsizes_x, cumsizes_y, intervals, ret)
```

Arguments

`x`, `y`	`matrix|data.frame()` Data frames corresponding to the input data of left and right sides, respectively. Each data frame has at least three columns, the last one is the set of each point. See also `read_datasets()`.
`cumsizes_x`, `cumsizes_y`	Cumulative size of the different sets of points in `x` and `y`.
`intervals`	`integer(1)` The absolute range of the differences $[0, 1]$ is partitioned into the number of intervals provided.
`ret`	`("points"|"rectangles"|"polygons")` The format of the returned EAF differences.

Value

With `rectangle=FALSE`, a `data.frame` containing points where there is a transition in the value of the EAF differences. With `rectangle=TRUE`, a `matrix` where the first 4 columns give the coordinates of two corners of each rectangle. In both cases, the last column gives the difference in terms of sets in `x` minus sets in `y` that attain each point (i.e., negative values are differences in favour `y`).

See Also

`as_double_matrix()` `transform_maximise()`

compute_eaf_call — *Same as* `eaf()` *but performs no checks and does not transform the input or the output. This function should be used by other packages that want to avoid redundant checks and transformations.*

Description

Same as `eaf()` but performs no checks and does not transform the input or the output. This function should be used by other packages that want to avoid redundant checks and transformations.

Usage

```
compute_eaf_call(x, cumsizes, percentiles)
```

Arguments

x	`matrix()|data.frame()` Matrix or data frame of numerical values, where each row gives the coordinates of a point. If `sets` is missing, the last column of `x` gives the sets.
cumsizes	`integer()` Cumulative size of the different sets of points in `x`.
percentiles	`numeric()` Vector indicating which percentiles are computed. `NULL` computes all.

Value

`data.frame()`
A data frame containing the exact representation of EAF. The last column gives the percentile that corresponds to each point. If groups is not `NULL`, then an additional column indicates to which group the point belongs.

See Also

`as_double_matrix()` `transform_maximise()`

CPFs — *Conditional Pareto fronts obtained from Gaussian processes simulations.*

Description

The data has the only goal of providing an example of use of `vorobT()` and `vorobDev()`. It has been obtained by fitting two Gaussian processes on 20 observations of a bi-objective problem, before generating conditional simulation of both GPs at different locations and extracting non-dominated values of coupled simulations.

Usage

```
CPFs
```

Format

A data frame with 2967 observations on the following 3 variables.

`f1` first objective values.

`f2` second objective values.

`set` indices of corresponding conditional Pareto fronts.

Source

M Binois, D Ginsbourger, O Roustant (2015). "Quantifying uncertainty on Pareto fronts with Gaussian process conditional simulations." *European Journal of Operational Research*, **243**(2), 386–394. doi: 10.1016/j.ejor.2014.07.032.

Examples

```
data(CPFs)
vorobT(CPFs, reference = c(2, 200))
```

---

eaf *Exact computation of the EAF in 2D or 3D*

Description

This function computes the EAF given a set of 2D or 3D points and a vector `set` that indicates to which set each point belongs.

Usage

```
eaf(x, sets, percentiles = NULL, maximise = FALSE, groups = NULL)
```

Arguments

`x`	`matrix()|data.frame()` Matrix or data frame of numerical values, where each row gives the coordinates of a point. If `sets` is missing, the last column of `x` gives the sets.
`sets`	`integer()` A vector that indicates the set of each point in `x`. If missing, the last column of `x` is used instead.
`percentiles`	`numeric()` Vector indicating which percentiles are computed. `NULL` computes all.

`maximise` `logical()`
Whether the objectives must be maximised instead of minimised. Either a single logical value that applies to all objectives or a vector of logical values, with one value per objective.

`groups` `factor()`
Indicates that the EAF must be computed separately for data belonging to different groups.

Value

`data.frame()`
A data frame containing the exact representation of EAF. The last column gives the percentile that corresponds to each point. If groups is not `NULL`, then an additional column indicates to which group the point belongs.

Note

There are several examples of data sets in `system.file(package="moocore","extdata")`. The current implementation only supports two and three dimensional points.

Author(s)

Manuel López-Ibáñez

References

Viviane Grunert da Fonseca, Carlos M. Fonseca, Andreia O. Hall (2001). "Inferential Performance Assessment of Stochastic Optimisers and the Attainment Function." In Eckart Zitzler, Kalyanmoy Deb, Lothar Thiele, Carlos A. Coello Coello, David Corne (eds.), *Evolutionary Multi-criterion Optimization, EMO 2001*, volume 1993 of *Lecture Notes in Computer Science*, 213–225. Springer, Berlin~/ Heidelberg. doi: 10.1007/3540447199_15.

Carlos M. Fonseca, Andreia P. Guerreiro, Manuel López-Ibáñez, Luís Paquete (2011). "On the Computation of the Empirical Attainment Function." In R H C Takahashi, Kalyanmoy Deb, Elizabeth F. Wanner, Salvatore Greco (eds.), *Evolutionary Multi-criterion Optimization, EMO 2011*, volume 6576 of *Lecture Notes in Computer Science*, 106–120. Springer, Berlin~/ Heidelberg. doi: 10.1007/9783642198939_8.

See Also

`read_datasets()`

Examples

```
extdata_path <- system.file(package="moocore", "extdata")

x <- read_datasets(file.path(extdata_path, "example1_dat"))
# Compute full EAF (sets is the last column)
str(eaf(x))

# Compute only best, median and worst
```

```
str(eaf(x[,1:2], sets = x[,3], percentiles = c(0, 50, 100)))

x <- read_datasets(file.path(extdata_path, "spherical-250-10-3d.txt"))
y <- read_datasets(file.path(extdata_path, "uniform-250-10-3d.txt"))
x <- rbind(data.frame(x, groups = "spherical"),
           data.frame(y, groups = "uniform"))
# Compute only median separately for each group
z <- eaf(x[,1:3], sets = x[,4], groups = x[,5], percentiles = 50)
str(z)
```

`eafdiff` *Compute empirical attainment function differences*

Description

Calculate the differences between the empirical attainment functions of two data sets.

Usage

```
eafdiff(x, y, intervals = NULL, maximise = FALSE, rectangles = FALSE)
```

Arguments

`x, y`	`matrix|data.frame()` Data frames corresponding to the input data of left and right sides, respectively. Each data frame has at least three columns, the last one is the set of each point. See also `read_datasets()`.
`intervals`	`integer(1)` The absolute range of the differences $[0, 1]$ is partitioned into the number of intervals provided.
`maximise`	`logical()` Whether the objectives must be maximised instead of minimised. Either a single logical value that applies to all objectives or a vector of logical values, with one value per objective.
`rectangles`	`logical(1)` If TRUE, the output is in the form of rectangles of the same color.

Details

This function calculates the differences between the EAFs of two data sets.

Value

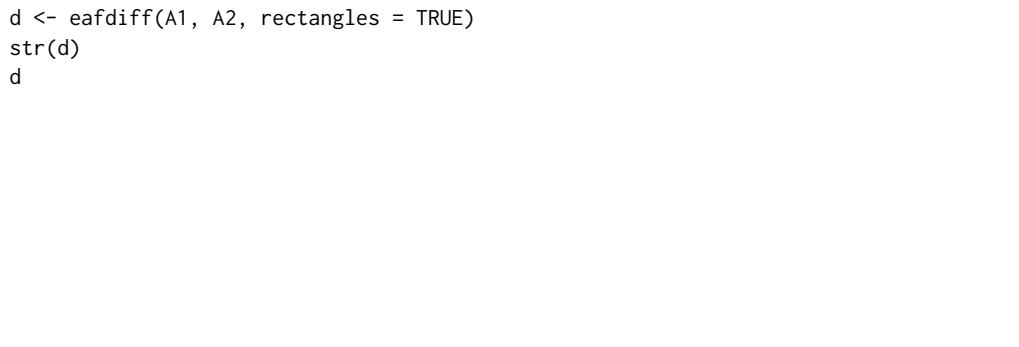
With `rectangle=FALSE`, a `data.frame` containing points where there is a transition in the value of the EAF differences. With `rectangle=TRUE`, a `matrix` where the first 4 columns give the coordinates of two corners of each rectangle. In both cases, the last column gives the difference in terms of sets in `x` minus sets in `y` that attain each point (i.e., negative values are differences in favour `y`).

See Also

read_datasets()

Examples

```
A1 <- read_datasets(text='
 3 2
 2 3

 2.5 1
 1 2

 1 2
')

A2 <- read_datasets(text='
 4 2.5
 3 3
 2.5 3.5

 3 3
 2.5 3.5

 2 1
')
d <- eafdiff(A1, A2)
str(d)
d
```



```
d <- eafdiff(A1, A2, rectangles = TRUE)
str(d)
d
```

eaf_as_list — *Convert an EAF data frame to a list of data frames, where each element of the list is one attainment surface. The function attsurf2df() can be used to convert the list into a single data frame.*

Description

Convert an EAF data frame to a list of data frames, where each element of the list is one attainment surface. The function attsurf2df() can be used to convert the list into a single data frame.

Usage

```
eaf_as_list(eaf)
```

Arguments

eaf	data.frame()|matrix() Data frame or matrix that represents the EAF.

Value

list()
A list of data frames. Each data.frame represents one attainment surface.

See Also

eaf() attsurf2df()

Examples

```
extdata_path <- system.file(package="moocore", "extdata")
x <- read_datasets(file.path(extdata_path, "example1_dat"))
attsurfs <- eaf_as_list(eaf(x, percentiles = c(0, 50, 100)))
str(attsurfs)
```

epsilon — *Epsilon metric*

Description

Computes the epsilon metric, either additive or multiplicative.

Usage

```
epsilon_additive(x, reference, maximise = FALSE)

epsilon_mult(x, reference, maximise = FALSE)
```

Arguments

x	`matrix()|data.frame()` Matrix or data frame of numerical values, where each row gives the coordinates of a point.
reference	`matrix|data.frame` Reference set as a matrix or data.frame of numerical values.
maximise	`logical()` Whether the objectives must be maximised instead of minimised. Either a single logical value that applies to all objectives or a vector of logical values, with one value per objective.

Details

The epsilon metric of a set A with respect to a reference set R is defined as

$$epsilon(A, R) = \max_{r \in R} \min_{a \in A} \max_{1 \leq i \leq n} epsilon(a_i, r_i)$$

where a and b are objective vectors and, in the case of minimization of objective i, $epsilon(a_i, b_i)$ is computed as a_i / b_i for the multiplicative variant (respectively, $a_i - b_i$ for the additive variant), whereas in the case of maximization of objective i, $epsilon(a_i, b_i) = b_i / a_i$ for the multiplicative variant (respectively, $b_i - a_i$ for the additive variant). This allows computing a single value for problems where some objectives are to be maximized while others are to be minimized. Moreover, a lower value corresponds to a better approximation set, independently of the type of problem (minimization, maximization or mixed). However, the meaning of the value is different for each objective type. For example, imagine that objective 1 is to be minimized and objective 2 is to be maximized, and the multiplicative epsilon computed here for $epsilon(A, R) = 3$. This means that A needs to be multiplied by 1/3 for all a_1 values and by 3 for all a_2 values in order to weakly dominate R. The computation of the multiplicative version for negative values doesn't make sense.

Computation of the epsilon indicator requires $O(n \cdot |A| \cdot |R|)$, where n is the number of objectives (dimension of vectors).

Value

`numeric(1)`
A single numerical value.

Author(s)

Manuel López-Ibáñez

References

Eckart Zitzler, Lothar Thiele, Marco Laumanns, Carlos M. Fonseca, Viviane Grunert da Fonseca (2003). "Performance Assessment of Multiobjective Optimizers: an Analysis and Review." *IEEE Transactions on Evolutionary Computation*, **7**(2), 117–132. doi: 10.1109/TEVC.2003.810758.

Examples

```
# Fig 6 from Zitzler et al. (2003).
A1 <- matrix(c(9,2,8,4,7,5,5,6,4,7), ncol=2, byrow=TRUE)
A2 <- matrix(c(8,4,7,5,5,6,4,7), ncol=2, byrow=TRUE)
A3 <- matrix(c(10,4,9,5,8,6,7,7,6,8), ncol=2, byrow=TRUE)
if (requireNamespace("graphics", quietly = TRUE)) {
   plot(A1, xlab=expression(f[1]), ylab=expression(f[2]),
        panel.first=grid(nx=NULL), pch=4, cex=1.5, xlim = c(0,10), ylim=c(0,8))
   points(A2, pch=0, cex=1.5)
   points(A3, pch=1, cex=1.5)
   legend("bottomleft", legend=c("A1", "A2", "A3"), pch=c(4,0,1),
          pt.bg="gray", bg="white", bty = "n", pt.cex=1.5, cex=1.2)
}
epsilon_mult(A1, A3) # A1 epsilon-dominates A3 => e = 9/10 < 1
epsilon_mult(A1, A2) # A1 weakly dominates A2 => e = 1
epsilon_mult(A2, A1) # A2 is epsilon-dominated by A1 => e = 2 > 1

# A more realistic example
extdata_path <- system.file(package="moocore","extdata")
path.A1 <- file.path(extdata_path, "ALG_1_dat.xz")
path.A2 <- file.path(extdata_path, "ALG_2_dat.xz")
A1 <- read_datasets(path.A1)[,1:2]
A2 <- read_datasets(path.A2)[,1:2]
ref <- filter_dominated(rbind(A1, A2))
epsilon_additive(A1, ref)
epsilon_additive(A2, ref)
# Multiplicative version of epsilon metric
ref <- filter_dominated(rbind(A1, A2))
epsilon_mult(A1, ref)
epsilon_mult(A2, ref)
```

`hv_contributions` *Hypervolume contribution of a set of points*

Description

Computes the hypervolume contribution of each point given a set of points with respect to a given reference point assuming minimization of all objectives. Dominated points have zero contribution. Duplicated points have zero contribution even if not dominated, because removing one of them does not change the hypervolume dominated by the remaining set.

Usage

```
hv_contributions(x, reference, maximise = FALSE)
```

Arguments

x	`matrix()`|`data.frame()` Matrix or data frame of numerical values, where each row gives the coordinates of a point.

`reference` `numeric()`
Reference point as a vector of numerical values.

`maximise` `logical()`
Whether the objectives must be maximised instead of minimised. Either a single logical value that applies to all objectives or a vector of logical values, with one value per objective.

Value

`numeric()`
A numerical vector

Author(s)

Manuel López-Ibáñez

References

Carlos M. Fonseca, Luís Paquete, Manuel López-Ibáñez (2006). "An improved dimension-sweep algorithm for the hypervolume indicator." In *Proceedings of the 2006 Congress on Evolutionary Computation (CEC 2006)*, 1157–1163. doi: 10.1109/CEC.2006.1688440.

Nicola Beume, Carlos M. Fonseca, Manuel López-Ibáñez, Luís Paquete, Jan Vahrenhold (2009). "On the complexity of computing the hypervolume indicator." *IEEE Transactions on Evolutionary Computation*, **13**(5), 1075–1082. doi: 10.1109/TEVC.2009.2015575.

See Also

`hypervolume()`

Examples

```
data(SPEA2minstoptimeRichmond)
# The second objective must be maximized
# We calculate the hypervolume contribution of each point of the union of all sets.
hv_contributions(SPEA2minstoptimeRichmond[, 1:2], reference = c(250, 0),
            maximise = c(FALSE, TRUE))

# Duplicated points show zero contribution above, even if not
# dominated. However, filter_dominated removes all duplicates except
# one. Hence, there are more points below with nonzero contribution.
hv_contributions(filter_dominated(SPEA2minstoptimeRichmond[, 1:2], maximise = c(FALSE, TRUE)),
                 reference = c(250, 0), maximise = c(FALSE, TRUE))
```

HybridGA *Results of Hybrid GA on Vanzyl and Richmond water networks*

Description

Results of Hybrid GA on Vanzyl and Richmond water networks

Usage

```
HybridGA
```

Format

A list with two data frames, each of them with three columns, as produced by `read_datasets()`.

`$vanzyl` data frame of results on Vanzyl network

`$richmond` data frame of results on Richmond network. The second column is filled with `NA`

Source

Manuel López-Ibáñez (2009). *Operational Optimisation of Water Distribution Networks*. Ph.D. thesis, School of Engineering and the Built Environment, Edinburgh Napier University, UK. https://lopez-ibanez.eu/publications#LopezIbanezPhD..

Examples

```
data(HybridGA)
print(HybridGA$vanzyl)
print(HybridGA$richmond)
```

hypervolume *Hypervolume metric*

Description

Compute the hypervolume metric with respect to a given reference point assuming minimization of all objectives. For 2D and 3D, the algorithm used (Fonseca et al. 2006; Beume et al. 2009) has $O(n \log n)$ complexity. For 4D or higher, the algorithm (Fonseca et al. 2006) has $O(n^{d-2} \log n)$ time and linear space complexity in the worst-case, but experimental results show that the pruning techniques used may reduce the time complexity even further.

Usage

```
hypervolume(x, reference, maximise = FALSE)
```

Arguments

x	`matrix()|data.frame()` Matrix or data frame of numerical values, where each row gives the coordinates of a point.
reference	`numeric()` Reference point as a vector of numerical values.
maximise	`logical()` Whether the objectives must be maximised instead of minimised. Either a single logical value that applies to all objectives or a vector of logical values, with one value per objective.

Value

`numeric(1)`
A single numerical value.

Author(s)

Manuel López-Ibáñez

References

Nicola Beume, Carlos M. Fonseca, Manuel López-Ibáñez, Luís Paquete, Jan Vahrenhold (2009). "On the complexity of computing the hypervolume indicator." *IEEE Transactions on Evolutionary Computation*, **13**(5), 1075–1082. doi: 10.1109/TEVC.2009.2015575.

Carlos M. Fonseca, Luís Paquete, Manuel López-Ibáñez (2006). "An improved dimension-sweep algorithm for the hypervolume indicator." In *Proceedings of the 2006 Congress on Evolutionary Computation (CEC 2006)*, 1157–1163. doi: 10.1109/CEC.2006.1688440.

Examples

```
data(SPEA2minstoptimeRichmond)
# The second objective must be maximized
# We calculate the hypervolume of the union of all sets.
hypervolume(SPEA2minstoptimeRichmond[, 1:2], reference = c(250, 0),
            maximise = c(FALSE, TRUE))
```

---

igd *Inverted Generational Distance (IGD and IGD+) and Averaged Hausdorff Distance*

Description

Functions to compute the inverted generational distance (IGD and IGD+) and the averaged Hausdorff distance between nondominated sets of points.

Usage

```
igd(x, reference, maximise = FALSE)

igd_plus(x, reference, maximise = FALSE)

avg_hausdorff_dist(x, reference, maximise = FALSE, p = 1L)
```

Arguments

x	`matrix()|data.frame()` Matrix or data frame of numerical values, where each row gives the coordinates of a point.
reference	`matrix|data.frame` Reference set as a matrix or data.frame of numerical values.
maximise	`logical()` Whether the objectives must be maximised instead of minimised. Either a single logical value that applies to all objectives or a vector of logical values, with one value per objective.
p	`integer(1)` Hausdorff distance parameter (default: `1L`).

Details

The generational distance (GD) of a set A is defined as the distance between each point $a \in A$ and the closest point r in a reference set R, averaged over the size of A. Formally,

$$GD_p(A, R) = \left(\frac{1}{|A|} \sum_{a \in A} \min_{r \in R} d(a, r)^p \right)^{\frac{1}{p}}$$

where the distance in our implementation is the Euclidean distance:

$$d(a, r) = \sqrt{\sum_{k=1}^{M} (a_k - r_k)^2}$$

The inverted generational distance (IGD) is calculated as $IGD_p(A, R) = GD_p(R, A)$.

The modified inverted generational distanced (IGD+) was proposed by Ishibuchi et al. (2015) to ensure that IGD+ is weakly Pareto compliant, similarly to `epsilon_additive()` or `epsilon_mult()`. It modifies the distance measure as:

$$d^+(r, a) = \sqrt{\sum_{k=1}^{M} (\max\{r_k - a_k, 0\})^2}$$

The average Hausdorff distance (Δ_p) was proposed by Schütze et al. (2012) and it is calculated as:

$$\Delta_p(A, R) = \max\{IGD_p(A, R), IGD_p(R, A)\}$$

IGDX (Zhou et al. 2009) is the application of IGD to decision vectors instead of objective vectors to measure closeness and diversity in decision space. One can use the functions `igd()` or `igd_plus()` (recommended) directly, just passing the decision vectors as `data`.

There are different formulations of the GD and IGD metrics in the literature that differ on the value of p, on the distance metric used and on whether the term $|A|^{-1}$ is inside (as above) or outside the exponent $1/p$. GD was first proposed by Van Veldhuizen and Lamont (1998) with $p = 2$ and the term $|A|^{-1}$ outside the exponent. IGD seems to have been mentioned first by Coello Coello and Reyes-Sierra (2004), however, some people also used the name D-metric for the same concept with $p = 1$ and later papers have often used IGD/GD with $p = 1$. Schütze et al. (2012) proposed to place the term $|A|^{-1}$ inside the exponent, as in the formulation shown above. This has a significant effect for GD and less so for IGD given a constant reference set. IGD+ also follows this formulation. We refer to Ishibuchi et al. (2015) and Bezerra et al. (2017) for a more detailed historical perspective and a comparison of the various variants.

Following Ishibuchi et al. (2015), we always use $p = 1$ in our implementation of IGD and IGD+ because (1) it is the setting most used in recent works; (2) it makes irrelevant whether the term $|A|^{-1}$ is inside or outside the exponent $1/p$; and (3) the meaning of IGD becomes the average Euclidean distance from each reference point to its nearest objective vector. It is also slightly faster to compute.

GD should never be used directly to compare the quality of approximations to a Pareto front, as it often contradicts Pareto optimality (it is not weakly Pareto-compliant). We recommend IGD+ instead of IGD, since the latter contradicts Pareto optimality in some cases (see examples below) whereas IGD+ is weakly Pareto-compliant, but we implement IGD here because it is still popular due to historical reasons.

The average Hausdorff distance ($\Delta_p(A, R)$) is also not weakly Pareto-compliant, as shown in the examples below.

Value

`numeric(1)`
A single numerical value.

Author(s)

Manuel López-Ibáñez

References

Leonardo C. T. Bezerra, Manuel López-Ibáñez, Thomas Stützle (2017). "An Empirical Assessment of the Properties of Inverted Generational Distance Indicators on Multi- and Many-objective Optimization." In Heike Trautmann, Günter Rudolph, Kathrin Klamroth, Oliver Schütze, Margaret M. Wiecek, Yaochu Jin, Christian Grimme (eds.), *Evolutionary Multi-criterion Optimization, EMO 2017*, volume 10173 of *Lecture Notes in Computer Science*, 31–45. Springer International Publishing, Cham, Switzerland. doi: 10.1007/9783319541570_3.

Carlos A. Coello Coello, Margarita Reyes-Sierra (2004). "A Study of the Parallelization of a Coevolutionary Multi-objective Evolutionary Algorithm." In Raúl Monroy, Gustavo Arroyo-Figueroa, Luis Enrique Sucar, Humberto Sossa (eds.), *Proceedings of MICAI*, volume 2972 of *Lecture Notes in Artificial Intelligence*, 688–697. Springer, Heidelberg, Germany.

Hisao Ishibuchi, Hiroyuki Masuda, Yuki Tanigaki, Yusuke Nojima (2015). "Modified Distance Calculation in Generational Distance and Inverted Generational Distance." In António Gaspar-Cunha, Carlos Henggeler Antunes, Carlos A. Coello Coello (eds.), *Evolutionary Multi-criterion Optimization, EMO 2015 Part I*, volume 9018 of *Lecture Notes in Computer Science*, 110–125. Springer, Heidelberg, Germany.

Oliver Schütze, X Esquivel, A Lara, Carlos A. Coello Coello (2012). "Using the Averaged Hausdorff Distance as a Performance Measure in Evolutionary Multiobjective Optimization." *IEEE Transactions on Evolutionary Computation*, **16**(4), 504–522.

David A. Van Veldhuizen, Gary B. Lamont (1998). "Evolutionary Computation and Convergence to a Pareto Front." In John R. Koza (ed.), *Late Breaking Papers at the Genetic Programming 1998 Conference*, 221–228.

A Zhou, Qingfu Zhang, Yaochu Jin (2009). "Approximating the set of Pareto-optimal solutions in both the decision and objective spaces by an estimation of distribution algorithm." *IEEE Transactions on Evolutionary Computation*, **13**(5), 1167–1189. doi: 10.1109/TEVC.2009.2021467.

Examples

```
# Example 4 from Ishibuchi et al. (2015)
ref <- matrix(c(10,0,6,1,2,2,1,6,0,10), ncol=2, byrow=TRUE)
A <- matrix(c(4,2,3,3,2,4), ncol=2, byrow=TRUE)
B <- matrix(c(8,2,4,4,2,8), ncol=2, byrow=TRUE)
if (requireNamespace("graphics", quietly = TRUE)) {
   plot(ref, xlab=expression(f[1]), ylab=expression(f[2]),
        panel.first=grid(nx=NULL), pch=23, bg="gray", cex=1.5)
   points(A, pch=1, cex=1.5)
   points(B, pch=19, cex=1.5)
   legend("topright", legend=c("Reference", "A", "B"), pch=c(23,1,19),
          pt.bg="gray", bg="white", bty = "n", pt.cex=1.5, cex=1.2)
}
cat("A is better than B in terms of Pareto optimality,\n however, IGD(A)=",
    igd(A, ref), "> IGD(B)=", igd(B, ref),
    "and AvgHausdorff(A)=", avg_hausdorff_dist(A, ref),
    "> AvgHausdorff(A)=", avg_hausdorff_dist(B, ref),
    ", which both contradict Pareto optimality.\nBy contrast, IGD+(A)=",
    igd_plus(A, ref), "< IGD+(B)=", igd_plus(B, ref), ", which is correct.\n")
# A less trivial example.
extdata_path <- system.file(package="moocore","extdata")
path.A1 <- file.path(extdata_path, "ALG_1_dat.xz")
path.A2 <- file.path(extdata_path, "ALG_2_dat.xz")
A1 <- read_datasets(path.A1)[,1:2]
A2 <- read_datasets(path.A2)[,1:2]
ref <- filter_dominated(rbind(A1, A2))
igd(A1, ref)
igd(A2, ref)

# IGD+ (Pareto compliant)
igd_plus(A1, ref)
```

```
igd_plus(A2, ref)

# Average Haussdorff distance
avg_hausdorff_dist(A1, ref)
avg_hausdorff_dist(A2, ref)
```

`is_nondominated` *Identify, remove and rank dominated points according to Pareto optimality*

Description

Identify nondominated points with `is_nondominated()` and remove dominated ones with `filter_dominated()`.

`pareto_rank()` ranks points according to Pareto-optimality, which is also called nondominated sorting (Deb et al. 2002).

Usage

```
is_nondominated(x, maximise = FALSE, keep_weakly = FALSE)

filter_dominated(x, maximise = FALSE, keep_weakly = FALSE)

pareto_rank(x, maximise = FALSE)
```

Arguments

`x`	`matrix()|data.frame()` Matrix or data frame of numerical values, where each row gives the coordinates of a point.
`maximise`	`logical()` Whether the objectives must be maximised instead of minimised. Either a single logical value that applies to all objectives or a vector of logical values, with one value per objective.
`keep_weakly`	If `FALSE`, return `FALSE` for any duplicates of nondominated points.

Details

`pareto_rank()` is meant to be used like `rank()`, but it assigns ranks according to Pareto dominance. Duplicated points are kept on the same front. When `ncol(data) == 2`, the code uses the $O(n \log n)$ algorithm by Jensen (2003).

Value

`is_nondominated()` returns a logical vector of the same length as the number of rows of `data`, where `TRUE` means that the point is not dominated by any other point.

`filter_dominated` returns a matrix or data.frame with only mutually nondominated points.

`pareto_rank()` returns an integer vector of the same length as the number of rows of `data`, where each value gives the rank of each point.

Author(s)

Manuel López-Ibáñez

References

Kalyanmoy Deb, A Pratap, S Agarwal, T Meyarivan (2002). "A fast and elitist multi-objective genetic algorithm: NSGA-II." *IEEE Transactions on Evolutionary Computation*, **6**(2), 182–197. doi: 10.1109/4235.996017.

M T Jensen (2003). "Reducing the run-time complexity of multiobjective EAs: The NSGA-II and other algorithms." *IEEE Transactions on Evolutionary Computation*, **7**(5), 503–515.

Examples

```
S = matrix(c(1,1,0,1,1,0,1,0), ncol = 2, byrow = TRUE)
is_nondominated(S)

is_nondominated(S, maximise = TRUE)

filter_dominated(S)

filter_dominated(S, keep_weakly = TRUE)

path_A1 <- file.path(system.file(package="moocore"),"extdata","ALG_1_dat.xz")
set <- read_datasets(path_A1)[,1:2]
is_nondom <- is_nondominated(set)
cat("There are ", sum(is_nondom), " nondominated points\n")

if (requireNamespace("graphics", quietly = TRUE)) {
   plot(set, col = "blue", type = "p", pch = 20)
   ndset <- filter_dominated(set)
   points(ndset[order(ndset[,1]),], col = "red", pch = 21)
}

ranks <- pareto_rank(set)
str(ranks)
if (requireNamespace("graphics", quietly = TRUE)) {
   colors <- colorRampPalette(c("red","yellow","springgreen","royalblue"))(max(ranks))
   plot(set, col = colors[ranks], type = "p", pch = 20)
}
```

---

largest_eafdiff *Identify largest EAF differences*

Description

Given a list of datasets, return the indexes of the pair with the largest EAF differences according to the method proposed by Diaz and López-Ibáñez (2021).

Usage

```
largest_eafdiff(x, maximise = FALSE, intervals = 5L, reference, ideal = NULL)
```

Arguments

`x`	`list()` A list of matrices or data frames with at least 3 columns (last column indicates the set).
`maximise`	`logical()` Whether the objectives must be maximised instead of minimised. Either a single logical value that applies to all objectives or a vector of logical values, with one value per objective.
`intervals`	`integer(1)` The absolute range of the differences $[0, 1]$ is partitioned into the number of intervals provided.
`reference`	`numeric()` Reference point as a vector of numerical values.
`ideal`	`numeric()` Ideal point as a vector of numerical values. If `NULL`, it is calculated as minimum (or maximum if maximising that objective) of each objective in the input data.

Value

`list()`
A list with two components `pair` and `value`.

References

Juan Esteban Diaz, Manuel López-Ibáñez (2021). "Incorporating Decision-Maker's Preferences into the Automatic Configuration of Bi-Objective Optimisation Algorithms." *European Journal of Operational Research*, **289**(3), 1209–1222. doi: 10.1016/j.ejor.2020.07.059.

Examples

```
# FIXME: This example is too large, we need a smaller one.
data(tpls50x20_1_MWT)
nadir <- apply(tpls50x20_1_MWT[,2:3], 2L, max)
x <- largest_eafdiff(split.data.frame(tpls50x20_1_MWT[,2:4], tpls50x20_1_MWT[, 1L]),
                     reference = nadir)
str(x)
```

normalise *Normalise points*

Description

Normalise points per coordinate to a range, e.g., `c(1,2)`, where the minimum value will correspond to 1 and the maximum to 2. If bounds are given, they are used for the normalisation.

Usage

```
normalise(x, to_range = c(1, 2), lower = NA, upper = NA, maximise = FALSE)
```

Arguments

`x`	`matrix()|data.frame()` Matrix or data frame of numerical values, where each row gives the coordinates of a point.
`to_range`	`numerical(2)` Normalise values to this range. If the objective is maximised, it is normalised to `c(to_range[1], to_range[0])` instead.
`lower, upper`	`numerical()` Bounds on the values. If `NA`, the maximum and minimum values of each coordinate are used.
`maximise`	`logical()` Whether the objectives must be maximised instead of minimised. Either a single logical value that applies to all objectives or a vector of logical values, with one value per objective.

Value

`matrix()`
A numerical matrix

Author(s)

Manuel López-Ibáñez

Examples

```
data(SPEA2minstoptimeRichmond)
# The second objective must be maximized
head(SPEA2minstoptimeRichmond[, 1:2])

head(normalise(SPEA2minstoptimeRichmond[, 1:2], maximise = c(FALSE, TRUE)))

head(normalise(SPEA2minstoptimeRichmond[, 1:2], to_range = c(0,1), maximise = c(FALSE, TRUE)))
```

rbind_datasets *Combine datasets* x *and* y *by row taking care of making all sets unique.*

Description

Combine datasets x and y by row taking care of making all sets unique.

Usage

```
rbind_datasets(x, y)
```

Arguments

x, y	matrix|data.frame() Each dataset has at least three columns, the last one is the set of each point. See also read_datasets().

Value

matrix()|data.frame()‘
A dataset.

Examples

```
x <- data.frame(f1 = 5:10, f2 = 10:5, set = 1:6)
y <- data.frame(f1 = 15:20, f2 = 20:15, set = 1:6)
rbind_datasets(x,y)
```

read_datasets *Read several data sets*

Description

Reads a text file in table format and creates a matrix from it. The file may contain several sets, separated by empty lines. Lines starting by '#' are considered comments and treated as empty lines. The function adds an additional column set to indicate to which set each row belongs.

Usage

```
read_datasets(file, col_names, text)
```

Arguments

file	character() Filename that contains the data. Each row of the table appears as one line of the file. If it does not contain an *absolute* path, the file name is *relative* to the current working directory, base::getwd(). Tilde-expansion is performed where supported. Files compressed with xz are supported.
col_names	character() Vector of optional names for the variables. The default is to use '"V"' followed by the column number.
text	character() If file is not supplied and this is, then data are read from the value of text via a text connection. Notice that a literal string can be used to include (small) data sets within R code.

Value

matrix()
A numerical matrix of the data in the file. An extra column set is added to indicate to which set each row belongs.

Warning

A known limitation is that the input file must use newline characters native to the host system, otherwise they will be, possibly silently, misinterpreted. In GNU/Linux the program dos2unix may be used to fix newline characters.

Note

There are several examples of data sets in system.file(package="moocore","extdata").

Author(s)

Manuel López-Ibáñez

See Also

utils::read.table()

Examples

```
extdata_path <- system.file(package="moocore","extdata")
A1 <- read_datasets(file.path(extdata_path,"ALG_1_dat.xz"))
str(A1)

read_datasets(text="1 2\n3 4\n\n5 6\n7 8\n", col_names=c("obj1", "obj2"))
```

SPEA2minstoptimeRichmond *Results of SPEA2 when minimising electrical cost and maximising the minimum idle time of pumps on Richmond water network.*

Description

Results of SPEA2 when minimising electrical cost and maximising the minimum idle time of pumps on Richmond water network.

Usage

```
SPEA2minstoptimeRichmond
```

Format

A data frame as produced by `read_datasets()`. The second column measures time in seconds and corresponds to a maximisation problem.

Source

Manuel López-Ibáñez (2009). *Operational Optimisation of Water Distribution Networks*. Ph.D. thesis, School of Engineering and the Built Environment, Edinburgh Napier University, UK. https://lopez-ibanez.eu/publications#LopezIbanezPhD.

Examples

```
data(SPEA2minstoptimeRichmond)
str(SPEA2minstoptimeRichmond)
```

SPEA2relativeRichmond *Results of SPEA2 with relative time-controlled triggers on Richmond water network.*

Description

Results of SPEA2 with relative time-controlled triggers on Richmond water network.

Usage

```
SPEA2relativeRichmond
```

Format

A data frame as produced by `read_datasets()`.

Source

Manuel López-Ibáñez (2009). *Operational Optimisation of Water Distribution Networks*. Ph.D. thesis, School of Engineering and the Built Environment, Edinburgh Napier University, UK. https://lopez-ibanez.eu/publications#LopezIbanezPhD.

Examples

```
data(SPEA2relativeRichmond)
str(SPEA2relativeRichmond)
```

SPEA2relativeVanzyl *Results of SPEA2 with relative time-controlled triggers on Vanzyl's water network.*

Description

Results of SPEA2 with relative time-controlled triggers on Vanzyl's water network.

Usage

```
SPEA2relativeVanzyl
```

Format

An object of class `data.frame` with 107 rows and 3 columns.

Source

Manuel López-Ibáñez (2009). *Operational Optimisation of Water Distribution Networks*. Ph.D. thesis, School of Engineering and the Built Environment, Edinburgh Napier University, UK. https://lopez-ibanez.eu/publications#LopezIbanezPhD.

Examples

```
data(SPEA2relativeVanzyl)
str(SPEA2relativeVanzyl)
```

tpls50x20_1_MWT *Various strategies of Two-Phase Local Search applied to the Permutation Flowshop Problem with Makespan and Weighted Tardiness objectives.*

Description

Various strategies of Two-Phase Local Search applied to the Permutation Flowshop Problem with Makespan and Weighted Tardiness objectives.

Usage

```
tpls50x20_1_MWT
```

Format

A data frame with 1511 observations of 4 variables:

`algorithm` TPLS search strategy

`Makespan` first objective values.

`WeightedTardiness` second objective values.

`set` indices of corresponding conditional Pareto fronts.

Source

Jérémie Dubois-Lacoste, Manuel López-Ibáñez, Thomas Stützle (2011). "Improving the Anytime Behavior of Two-Phase Local Search." *Annals of Mathematics and Artificial Intelligence*, **61**(2), 125–154. doi: 10.1007/s1047201192350.

Examples

```
data(tpls50x20_1_MWT)
str(tpls50x20_1_MWT)
```

transform_maximise *Transform matrix according to maximise parameter*

Description

Transform matrix according to maximise parameter

Usage

```
transform_maximise(x, maximise)
```

Arguments

x	matrix()|data.frame() Matrix or data frame of numerical values, where each row gives the coordinates of a point.
maximise	logical() Whether the objectives must be maximised instead of minimised. Either a single logical value that applies to all objectives or a vector of logical values, with one value per objective.

Value

x transformed such that every column where maximise is TRUE is multiplied by -1.

Examples

```
x <- data.frame(f1=1:10, f2=101:110)
rownames(x) <- letters[1:10]
transform_maximise(x, maximise=c(FALSE,TRUE))
transform_maximise(x, maximise=TRUE)
x <- as.matrix(x)
transform_maximise(x, maximise=c(FALSE,TRUE))
transform_maximise(x, maximise=TRUE)
```

---

vorobT *Vorob'ev computations*

Description

Compute Vorob'ev threshold, expectation and deviation. Also, displaying the symmetric deviation function is possible. The symmetric deviation function is the probability for a given target in the objective space to belong to the symmetric difference between the Vorob'ev expectation and a realization of the (random) attained set.

Usage

```
vorobT(x, sets, reference, maximise = FALSE)

vorobDev(x, sets, reference, VE = NULL, maximise = FALSE)
```

Arguments

x	matrix()|data.frame() Matrix or data frame of numerical values, where each row gives the coordinates of a point. If sets is missing, the last column of x gives the sets.
sets	integer() A vector that indicates the set of each point in x. If missing, the last column of x is used instead.

reference	numeric() Reference point as a vector of numerical values.
maximise	logical() Whether the objectives must be maximised instead of minimised. Either a single logical value that applies to all objectives or a vector of logical values, with one value per objective.
VE	matrix() Vorob'ev expectation, e.g., as returned by vorobT().

Value

vorobT returns a list with elements threshold, VE, and avg_hyp (average hypervolume)

vorobDev returns the Vorob'ev deviation.

Author(s)

Mickael Binois

References

M Binois, D Ginsbourger, O Roustant (2015). "Quantifying uncertainty on Pareto fronts with Gaussian process conditional simulations." *European Journal of Operational Research*, **243**(2), 386–394. doi: 10.1016/j.ejor.2014.07.032.

C. Chevalier (2013), Fast uncertainty reduction strategies relying on Gaussian process models, University of Bern, PhD thesis.

Ilya Molchanov (2005). *Theory of Random Sets*. Springer.

Examples

```
data(CPFs)
res <- vorobT(CPFs, reference = c(2, 200))
res$threshold
res$avg_hyp
# Now print Vorob'ev deviation
VD <- vorobDev(CPFs, VE = res$VE, reference = c(2, 200))
VD
```

---

whv_hype *Approximation of the (weighted) hypervolume by Monte-Carlo sampling (2D only)*

Description

Return an estimation of the hypervolume of the space dominated by the input data following the procedure described by Auger et al. (2009). A weight distribution describing user preferences may be specified.

Usage

```
whv_hype(
  x,
  reference,
  ideal,
  maximise = FALSE,
  dist = "uniform",
  nsamples = 100000L,
  seed = NULL,
  mu = NULL
)
```

Arguments

x	matrix()|data.frame() Matrix or data frame of numerical values, where each row gives the coordinates of a point.
reference	numeric() Reference point as a vector of numerical values.
ideal	numeric() Ideal point as a vector of numerical values.
maximise	logical() Whether the objectives must be maximised instead of minimised. Either a single logical value that applies to all objectives or a vector of logical values, with one value per objective.
dist	character(1) weight distribution type. See Details.
nsamples	integer(1) number of samples for Monte-Carlo sampling.
seed	integer(1) random seed.
mu	numeric() parameter of the weight distribution. See Details.

Details

The current implementation only supports 2 objectives.

A weight distribution (Auger et al. 2009) can be provided via the `dist` argument. The ones currently supported are:

- `"uniform"` corresponds to the default hypervolume (unweighted).
- `"point"` describes a goal in the objective space, where the parameter `mu` gives the coordinates of the goal. The resulting weight distribution is a multivariate normal distribution centred at the goal.
- `"exponential"` describes an exponential distribution with rate parameter `1/mu`, i.e., $\lambda = \frac{1}{\mu}$.

Value

A single numerical value.

References

Anne Auger, Johannes Bader, Dimo Brockhoff, Eckart Zitzler (2009). "Articulating User Preferences in Many-Objective Problems by Sampling the Weighted Hypervolume." In Franz Rothlauf (ed.), *Proceedings of the Genetic and Evolutionary Computation Conference, GECCO 2009*, 555–562. ACM Press, New York, NY.

See Also

`read_datasets()`, `eafdiff()`, `whv_rect()`

Examples

```
whv_hype(matrix(2, ncol=2), reference = 4, ideal = 1, seed = 42)
whv_hype(matrix(c(3,1), ncol=2), reference = 4, ideal = 1, seed = 42)
whv_hype(matrix(2, ncol=2), reference = 4, ideal = 1, seed = 42,
         dist = "exponential", mu=0.2)
whv_hype(matrix(c(3,1), ncol=2), reference = 4, ideal = 1, seed = 42,
         dist = "exponential", mu=0.2)
whv_hype(matrix(2, ncol=2), reference = 4, ideal = 1, seed = 42,
         dist = "point", mu=c(2.9,0.9))
whv_hype(matrix(c(3,1), ncol=2), reference = 4, ideal = 1, seed = 42,
         dist = "point", mu=c(2.9,0.9))
```

---

whv_rect *Compute (total) weighted hypervolume given a set of rectangles*

Description

Calculates the hypervolume weighted by a set of rectangles (with zero weight outside the rectangles). The function `total_whv_rect()` calculates the total weighted hypervolume as `hypervolume()` + `scalefactor` * `abs(prod(reference - ideal))` * `whv_rect()`. The details of the computation are given by Diaz and López-Ibáñez (2021).

Usage

```
whv_rect(x, rectangles, reference, maximise = FALSE)

total_whv_rect(
  x,
  rectangles,
  reference,
  maximise = FALSE,
  ideal = NULL,
  scalefactor = 0.1
)
```

Arguments

x	`matrix()|data.frame()` Matrix or data frame of numerical values, where each row gives the coordinates of a point.
rectangles	`matrix()` Weighted rectangles that will bias the computation of the hypervolume. Maybe generated by `eafdiff()` with `rectangles=TRUE` or by `choose_eafdiff()`.
reference	`numeric()` Reference point as a vector of numerical values.
maximise	`logical()` Whether the objectives must be maximised instead of minimised. Either a single logical value that applies to all objectives or a vector of logical values, with one value per objective.
ideal	`numeric()` Ideal point as a vector of numerical values. If `NULL`, it is calculated as minimum (or maximum if maximising that objective) of each objective in the input data.
scalefactor	`numeric(1)` Real value within $(0, 1]$ that scales the overall weight of the differences. This is parameter psi (ψ) in Diaz and López-Ibáñez (2021).

Details

TODO

Value

`numeric(1)`
A single numerical value.

References

Juan Esteban Diaz, Manuel López-Ibáñez (2021). "Incorporating Decision-Maker's Preferences into the Automatic Configuration of Bi-Objective Optimisation Algorithms." *European Journal of Operational Research*, **289**(3), 1209–1222. doi: 10.1016/j.ejor.2020.07.059.

See Also

`read_datasets()`, `eafdiff()`, `choose_eafdiff()`, `whv_hype()`

Examples

```
rectangles <- as.matrix(read.table(header=FALSE, text='
 1.0  3.0  2.0  Inf    1
 2.0  3.5  2.5  Inf    2
 2.0  3.0  3.0  3.5    3
'))
whv_rect (matrix(2, ncol=2), rectangles, reference = 6)
whv_rect (matrix(c(2, 1), ncol=2), rectangles, reference = 6)
```

```
whv_rect (matrix(c(1, 2), ncol=2), rectangles, reference = 6)

total_whv_rect (matrix(2, ncol=2), rectangles, reference = 6, ideal = c(1,1))
total_whv_rect (matrix(c(2, 1), ncol=2), rectangles, reference = 6, ideal = c(1,1))
total_whv_rect (matrix(c(1, 2), ncol=2), rectangles, reference = 6, ideal = c(1,1))
```

`write_datasets` *Write data sets*

Description

Write data sets to a file in the same format as `read_datasets()`.

Usage

```
write_datasets(x, file = "")
```

Arguments

`x`	`matrix|data.frame()` Dataset with at least three columns, the last one is the set of each point. See also `read_datasets()`.
`file`	Either a character string naming a file or a connection open for writing. `""` indicates output to the console.

Value

No return value, called for side effects

See Also

`utils::write.table()`, `read_datasets()`

Examples

```
x <- read_datasets(text="1 2\n3 4\n\n5 6\n7 8\n", col_names=c("obj1", "obj2"))
write_datasets(x)
```

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