

Package ‘nhppp’

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Title Simulating Nonhomogeneous Poisson Point Processes

Version 1.0.0

Description Simulates events from one dimensional nonhomogeneous Poisson point processes (NH-PPPs) as per Trikalinos and Sereda (2024, <[doi:10.48550/arXiv.2402.00358](https://doi.org/10.48550/arXiv.2402.00358)>). Functions are based on three algorithms that provably sample from a target NHPPP: the time-transformation of a homogeneous Poisson process (of intensity one) via the inverse of the integrated intensity function (Cinlar E, ``Theory of stochastic processes" (1975, ISBN:0486497996)); the generation of a Poisson number of order statistics from a fixed density function; and the thinning of a majorizing NHPPP via an acceptance-rejection scheme (Lewis PAW, Shedler, GS (1979) <[doi:10.1002/nav.3800260304](https://doi.org/10.1002/nav.3800260304)>).

License GPL (>= 3)

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Author Thomas Trikalinos [aut, cre, cph]
(<<https://orcid.org/0000-0002-3990-1848>>),
Yuliia Sereda [aut] (<<https://orcid.org/0000-0002-4017-4561>>)

Maintainer Thomas Trikalinos <thomas_trikalinos@brown.edu>

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draw	<i>Generic function for simulating from NHPPs given the intensity function or the cumulative intensity function.</i>
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Description

This is a wrapper to the package's specific functions, and thus somewhat slower. For time-intensive simulations prefer one of the specific functions.

Usage

```
draw(
  Lambda = NULL,
  Lambda_inv = NULL,
  lambda = NULL,
  line_majorizer_intercept = NULL,
  line_majorizer_slope = NULL,
  line_majorizer_is_loglinear = FALSE,
  step_majorizer_vector = NULL,
  t_min = NULL,
```

```

    t_max = NULL,
    atleast1 = FALSE,
    atleast1 = FALSE
)

```

Arguments

Lambda	(function, double vector) the integrated (cumulative) rate of the NHPPP
Lambda_inv	(function, double vector) the inverse of 'Lambda()'
lambda	(function) the instantaneous rate
line_majorizer_intercept	The intercept alpha of the loglinear majorizer function: $\alpha + \beta * t$ or $\exp(\alpha + \beta * t)$
line_majorizer_slope	The slope beta of the loglinear majorizer function: $\alpha + \beta * t$ or $\exp(\alpha + \beta * t)$
line_majorizer_is_loglinear	(boolean) if TRUE the majorizer is loglinear $\exp(\alpha + \beta * t)$; if FALSE it is a linear function
step_majorizer_vector	(vector, double) K constant majorizing rates, one per interval; all intervals are of equal length (regular)
t_min	(double) the lower bound of the interval
t_max	(double) the upper bound of the interval
atmost1	boolean, draw at most 1 event time
atleast1	boolean, draw at least 1 event time in interval

Value

a vector of event times

draw_cumulative_intensity

Simulate from a non homogeneous Poisson Point Process (NHPPP) over an interval when you know the cumulative intensity and its inverse.

Description

Sample NHPPP times using the inversion method

Usage

```
draw_cumulative_intensity(Lambda, Lambda_inv, t_min, t_max, atleast1 = FALSE)
```

Arguments

Lambda	(function, double vector) a continuous increasing R to R map which is the integrated rate of the NHPPP
Lambda_inv	(function, double vector) the inverse of Lambda()
t_min	(double) the lower bound of the time interval
t_max	(double) the upper bound of the time interval
atmost1	boolean, draw at most 1 event time

Value

a vector of event times (t_); if no events realize, a vector of length 0

draw_intensity	<i>Generic function for simulating from NHPPs given the intensity function.</i>
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Description

Sample from NHPPs given the intensity function This is a wrapper to the package's specific functions, and thus somewhat slower. For time-intensive simulations prefer one of the specific functions.

Usage

```
draw_intensity(
  lambda,
  line_majorizer_intercept = NULL,
  line_majorizer_slope = NULL,
  line_majorizer_is_loglinear = FALSE,
  step_majorizer_vector = NULL,
  t_min = NULL,
  t_max = NULL,
  atmost1 = FALSE
)
```

Arguments

lambda	(function) the instantaneous rate
line_majorizer_intercept	The intercept alpha of the loglinear majorizer function: $\alpha + \beta * t$ or $\exp(\alpha + \beta * t)$
line_majorizer_slope	The slope beta of the loglinear majorizer function: $\alpha + \beta * t$ or $\exp(\alpha + \beta * t)$
line_majorizer_is_loglinear	(boolean) if TRUE the majorizer is loglinear $\exp(\alpha + \beta * t)$; if FALSE it is a linear function

step_majorizer_vector	(vector, double) K constant majorizing rates, one per interval; all intervals are of equal length (regular)
t_min	(double) the lower bound of the interval
t_max	(double) the upper bound of the interval
atmost1	boolean, draw at most 1 event time

Value

a vector of event times

draw_sc_linear	<i>Special case: Simulate from a non homogeneous Poisson Point Process (NHPPP) from (t_min, t_max) with linear intensity function (inversion method)</i>
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Description

Sample NHPPP times from a linear intensity function using the inversion method, optionally using an rstream generator

Usage

```
draw_sc_linear(intercept, slope, t_min, t_max, atmost1 = FALSE)
```

Arguments

intercept	(double) the intercept
slope	(double) the slope
t_min	(double) lower bound of the time interval
t_max	(double) upper bound of the time interval
atmost1	boolean, draw at most 1 event time

Value

a vector of event times (t_); if no events realize, a vector of length 0

Examples

```
x <- draw_sc_linear(intercept = 0, slope = 0.2, t_min = 0, t_max = 10)
```

draw_sc_loglinear	<i>Special case: Simulate from a non homogeneous Poisson Point Process (NHPPP) from (t_min, t_max) with log-linear intensity function (inversion method)</i>
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Description

Sample NHPPP times from an log linear intensity function using the inversion method, optionally using an `rstream` generator

Usage

```
draw_sc_loglinear(intercept, slope, t_min, t_max, atmost1 = FALSE)
```

Arguments

<code>intercept</code>	(double) the intercept in the exponent
<code>slope</code>	(double) the slope in the exponent
<code>t_min</code>	(double) lower bound of the time interval
<code>t_max</code>	(double) upper bound of the time interval
<code>atmost1</code>	boolean, draw at most 1 event time

Value

a vector of event times (`t_`); if no events realize, a vector of length 0

Examples

```
x <- draw_sc_loglinear(intercept = 0, slope = 0.2, t_min = 0, t_max = 10)
```

draw_sc_step	<i>Simulate a piecewise constant-rate Poisson Point Process over (t_min, t_max] (inversion method) The intervals need not have the same length.</i>
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Description

Simulate a piecewise constant-rate Poisson Point Process over (`t_min`, `t_max`] (inversion method)
The intervals need not have the same length.

Usage

```
draw_sc_step(lambda_vector, time_breaks, atmost1 = FALSE, atleast1 = FALSE)
```

Arguments

lambda_vector	(scalar, double) K constant rates, one per interval
time_breaks	(vector, double) K+1 time points defining K intervals of constant rates: [t_1 = range_t[1], t_2): the first interval [t_k, t_{k+1}): the k-th interval [t_{K}, t_{K+1} = range_t[2]): the K-th (last) interval
atmost1	boolean, draw at most 1 event time
atleast1	boolean, draw at least 1 event time

Value

a vector of event times t if no events realize, it will have 0 length

Examples

```
x <- draw_sc_step(lambda_vector = rep(1, 5), time_breaks = c(0:5))
```

draw_sc_step_regular *Sampling from NHPPs with piecewise constant intensities with same interval lengths (non-vectorized)*

Description

Sampling from NHPPs with piecewise constant intensities with same interval lengths (non-vectorized)

Usage

```
draw_sc_step_regular(
  Lambda_vector = NULL,
  lambda_vector = NULL,
  t_min = NULL,
  t_max = NULL,
  atmost1 = FALSE,
  atleast1 = FALSE
)
```

Arguments

Lambda_vector	(scalar, double) K integrated intensity rates at the end of each interval
lambda_vector	(scalar, double) K constant intensity rates, one per interval
t_min	(scalar, double) lower bound of the time interval
t_max	(scalar, double) upper bound of the time interval
atmost1	boolean, draw at most 1 event time
atleast1	boolean, draw at least 1 event time

Value

a vector of event times t if no events realize, it will have 0 length

Examples

```
x <- draw_sc_step_regular(Lambda_vector = 1:5, t_min = 0, t_max = 5)
```

get_step_majorizer	<i>Piecewise constant (step) majorizer for K-Lipschitz functions over an interval (vectorized over the breaks argument).</i>
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Description

Return a piecewise constant (step) majorizer for K-Lipschitz functions over an interval. The function is vectorized over the breaks argument.

Usage

```
get_step_majorizer(fun, breaks, is_monotone = TRUE, K = 0)
```

Arguments

fun	A function object with a single argument x . If x is a matrix, fun should be vectorized over it.
breaks	(vector or matrix) The set of $M+1$ boundaries for the M subintervals in x . If breaks is a matrix, each row is treated as a separate set of breaks.
is_monotone	(boolean) Is the function monotone? (Default is TRUE.)
K	(double) A non-negative number for the Lipschitz cone. (Default is 0.)

Value

A vector of length M with the values of the piecewise constant majorizer

Examples

```
get_step_majorizer(fun = abs, breaks = -5:5, is_monotone = FALSE, K = 1)
```

ppp *Simulate a homogeneous Poisson Point Process in (t_min, t_max]*

Description

Simulate a homogeneous Poisson Point Process in (t_min, t_max]

Usage

```
ppp(rate, t_min, t_max, atmost1 = FALSE, tol = 10^-6)
```

Arguments

rate	(scalar, double) constant instantaneous rate
t_min	(scalar, double) the lower bound of the time interval
t_max	(scalar, double) the upper bound of the time interval
atmost1	boolean, draw at most 1 event time
tol	the probability that we will have more than the drawn events in (t_min, t_max]

Value

a vector of event times t if no events realize, it will have 0 length

Examples

```
x <- ppp(rate = 1, t_min = 0, t_max = 10, tol = 10^-6)
```

ppp_exactly_n *Simulate exactly n points from a homogeneous Poisson Point Process over (t_min, t_max]*

Description

Simulate exactly n points from a homogeneous Poisson Point Process over (t_min, t_max]

Usage

```
ppp_exactly_n(n, t_min, t_max)
```

Arguments

n	(int) the number of points to be simulated
t_min	(double) the lower bound of the time interval
t_max	(double) the upper bound of the time interval

Value

a vector of event times of size n

Examples

```
x <- ppp_exactly_n(n = 10, t_min = 0, t_max = 10)
```

ppp_next_n

Simulate n events from a homogeneous Poisson Point Process.

Description

Simulate n events from a homogeneous Poisson Point Process.

Usage

```
ppp_next_n(n = 1, rate = 1, t_min = 0, rng_stream = deprecated())
```

Arguments

n	scalar number of samples
rate	scalar instantaneous rate
t_min	scalar for the starting time value
rng_stream	[Deprecated] an rstream object

Value

a vector with event times t (starting from t_min)

Examples

```
x <- ppp_next_n(n = 10, rate = 1, t_min = 0)
```

vdraw	<i>Vectorized generic function for simulating from NHPPPs given the intensity function or the cumulative intensity function</i>
-------	---------------------------------------------------------------------------------------------------------------------------------

Description

This is a wrapper to the package's specific functions, and thus slightly slower. For time-intensive simulations prefer one of the specific functions.

Usage

```
vdraw(
  lambda = NULL,
  lambda_args = NULL,
  Lambda_maj_matrix = NULL,
  lambda_maj_matrix = NULL,
  Lambda = NULL,
  Lambda_inv = NULL,
  Lambda_args = NULL,
  Lambda_inv_args = NULL,
  t_min = NULL,
  t_max = NULL,
  rate_matrix_t_min = NULL,
  rate_matrix_t_max = NULL,
  tol = 10^-6,
  atmost1 = FALSE,
  atleast1 = FALSE,
  atmostB = NULL
)
```

Arguments

lambda	(function) intensity function, vectorized
lambda_args	(list) optional arguments to pass to lambda
Lambda_maj_matrix	(matrix) integrated intensity rates at the end of each interval
lambda_maj_matrix	(matrix) intensity rates, one per interval
Lambda	(function, double vector) an increasing function which is the integrated rate of the NHPPP. It should take a vectorized argument t for times and an optional arguments list.
Lambda_inv	(function, double vector) the inverse of Lambda(), also in vectorized form It should take a vectorized argument z and an optional arguments list.
Lambda_args	(list) optional arguments to pass to Lambda.

<code>Lambda_inv_args</code>	(list) optional arguments to pass to <code>Lambda_inv()</code> .
<code>t_min</code>	(scalar vector column matrix) is the lower bound of a subinterval of <code>(rate_matrix_t_min, rate_matrix_t_max]</code> . If set, times are sampled from the subinterval. If omitted, it is equivalent to <code>rate_matrix_t_min</code> .
<code>t_max</code>	(scalar vector column matrix) is the upper bound of a subinterval of <code>(rate_matrix_t_min, rate_matrix_t_max]</code> . If set, times are sampled from the subinterval. If omitted, it is equivalent to <code>rate_matrix_t_max</code> .
<code>rate_matrix_t_min</code>	(scalar vector column matrix) is the lower bound of the time interval for each row of <code>(Lambdallambda)_maj_matrix</code> . The length of this argument is the number of point processes that should be drawn.
<code>rate_matrix_t_max</code>	(scalar vector column matrix) the upper bound of the time interval for each row of <code>(Lambdallambda)_maj_matrix</code> . The length of this argument is the number of point processes that should be drawn.
<code>tol</code>	(scalar, double) tolerance for the number of events
<code>atmost1</code>	boolean, draw at most 1 event time
<code>atleast1</code>	boolean, draw at least 1 event time
<code>atmostB</code>	If not NULL, draw at most B (B>0) event times. NULL means ignore.

Value

a vector of event times

`vdraw_cumulative_intensity`

Vectorized simulation from a non homogeneous Poisson Point Process (NHPPP) from (t_{min}, t_{max}) given the cumulative intensity function and its inverse

Description

Sample NHPPP times using the cumulative intensity function and its inverse.

Usage

```
vdraw_cumulative_intensity(
  Lambda,
  Lambda_inv,
  t_min,
  t_max,
  Lambda_args = NULL,
  Lambda_inv_args = NULL,
  tol = 10^-6,
  atmost1 = FALSE,
  atleast1 = FALSE
)
```

Arguments

Lambda	(function, double vector) an increasing function which is the integrated rate of the NHPPP. It should take a vectorized argument t for times and an optional arguments list.
Lambda_inv	(function, double vector) the inverse of Lambda(), also in vectorized form It should take a vectorized argument z and an optional arguments list.
t_min	(scalar vector column matrix) the lower bound of the interval for each sampled point process The length of this argument is the number of point processes that should be drawn.
t_max	(scalar vector column matrix) the upper bound of the interval for each sampled point process The length of this argument is the number of point processes that should be drawn.
Lambda_args	(list) optional arguments to pass to Lambda.
Lambda_inv_args	(list) optional arguments to pass to Lambda_inv().
tol	the tolerance for the calculations.
atmost1	boolean, draw at most 1 event time per sampled point process.
atleast1	boolean, draw at least 1 event time

Value

a matrix of event times with one row per sampled point process.

vdraw_intensity	<i>Vectorized sampling from a non homogeneous Poisson Point Process (NHPPP) from an interval (thinning method) with piecewise constant majorizers (C++)</i>
-----------------	-------------------------------------------------------------------------------------------------------------------------------------------------------------

Description

Vectorized sampling from a non homogeneous Poisson Point Process (NHPPP) from an interval (thinning method) with piecewise constant majorizers. The majorizers are step functions over equal-length time intervals.

Usage

```
vdraw_intensity(
  lambda = NULL,
  lambda_args = NULL,
  Lambda_maj_matrix = NULL,
  lambda_maj_matrix = NULL,
  rate_matrix_t_min = NULL,
  rate_matrix_t_max = NULL,
  t_min = NULL,
```

```

    t_max = NULL,
    tol = 10^-6,
    atmost1 = FALSE,
    atleast1 = FALSE,
    atmostB = NULL
  )

```

Arguments

lambda (function) intensity function, vectorized

lambda_args (list) optional arguments to pass to lambda

Lambda_maj_matrix (matrix) integrated intensity rates at the end of each interval

lambda_maj_matrix (matrix) intensity rates, one per interval

rate_matrix_t_min (scalar | vector | column matrix) is the lower bound of the time interval for each row of (Lambdallambda)_maj_matrix. The length of this argument is the number of point processes that should be drawn.

rate_matrix_t_max (scalar | vector | column matrix) the upper bound of the time interval for each row of (Lambdallambda)_maj_matrix. The length of this argument is the number of point processes that should be drawn.

t_min (scalar | vector | column matrix) is the lower bound of a subinterval of (rate_matrix_t_min, rate_matrix_t_max]. If set, times are sampled from the subinterval. If omitted, it is equivalent to rate_matrix_t_min.

t_max (scalar | vector | column matrix) is the upper bound of a subinterval of (rate_matrix_t_min, rate_matrix_t_max]. If set, times are sampled from the subinterval. If omitted, it is equivalent to rate_matrix_t_max.

tol (scalar, double) tolerance for the number of events

atmost1 boolean, draw at most 1 event time

atleast1 boolean, draw at least 1 event time

atmostB If not NULL, draw at most B (B>0) event times. NULL means ignore.

Value

a matrix of event times (columns) per draw (rows) NAs are structural empty spots

Examples

```

x <- vdraw_intensity(
  lambda = function(x, ...) 0.1 * x,
  lambda_maj_matrix = matrix(rep(1, 5), nrow = 1),
  rate_matrix_t_min = 1,
  rate_matrix_t_max = 5
)

```

vdraw_sc_step_regular *Vectorized sampling from NHPPs with piecewise constant intensities with same interval lengths*

Description

Simulate a piecewise constant-rate Poisson Point Process over $(t_{\min}, t_{\max}]$ (inversion method) where the intervals have the same length (are "regular").

Usage

```
vdraw_sc_step_regular(
  lambda_matrix = NULL,
  Lambda_matrix = NULL,
  rate_matrix_t_min = NULL,
  rate_matrix_t_max = NULL,
  t_min = NULL,
  t_max = NULL,
  tol = 10^-6,
  atmost1 = FALSE,
  atmostB = NULL,
  atleast1 = FALSE
)
```

Arguments

lambda_matrix (matrix) intensity rates, one per interval

Lambda_matrix (matrix) integrated intensity rates at the end of each interval

rate_matrix_t_min
(scalar | vector | column matrix) is the lower bound of the time interval for each row of $(\text{Lambdallambda})_{\text{maj_matrix}}$. The length of this argument is the number of point processes that should be drawn.

rate_matrix_t_max
(scalar | vector | column matrix) the upper bound of the time interval for each row of $(\text{Lambdallambda})_{\text{maj_matrix}}$. The length of this argument is the number of point processes that should be drawn.

t_min (scalar | vector | column matrix) is the lower bound of a subinterval of $(\text{rate_matrix_t_min}, \text{rate_matrix_t_max}]$. If set, times are sampled from the subinterval. If omitted, it is equivalent to rate_matrix_t_min .

t_max (scalar | vector | column matrix) is the upper bound of a subinterval of $(\text{rate_matrix_t_min}, \text{rate_matrix_t_max}]$. If set, times are sampled from the subinterval. If omitted, it is equivalent to rate_matrix_t_max .

tol (scalar, double) tolerance for the number of events

atmost1 boolean, draw at most 1 event time

atmostB If not NULL, draw at most B ($B > 0$) event times. NULL means ignore.

atleast1 boolean, draw at least 1 event time

Value

a vector of event times t if no events realize, it will have 0 length

Examples

```
x <- vdraw_sc_step_regular(
  Lambda_matrix = matrix(1:5, nrow = 1),
  rate_matrix_t_min = 100,
  rate_matrix_t_max = 110,
  atmost1 = FALSE
)
```

ztdraw_cumulative_intensity

Simulate from a zero-truncated non homogeneous Poisson Point Process (zt-NHPPP) from (t_min, t_max) (order statistics method)

Description

Sample zero-truncated NHPPP times using the order statistics method, optionally using an `rstream` generator

Usage

```
ztdraw_cumulative_intensity(Lambda, Lambda_inv, t_min, t_max, atmost1 = FALSE)
```

Arguments

<code>Lambda</code>	(function, double vector) a continuous increasing \mathbb{R} to \mathbb{R} map which is the integrated rate of the NHPPP
<code>Lambda_inv</code>	(function, double vector) the inverse of <code>Lambda()</code>
<code>t_min</code>	(double) the lower bound of the time interval
<code>t_max</code>	(double) the upper bound of the time interval
<code>atmost1</code>	(boolean) draw at most 1 event time

Value

a vector of at least 1 event times

ztdraw_sc_linear	<i>Simulate size samples from a zero-truncated non homogeneous Poisson Point Process (zt-NHPPP) from (t_min, t_max) with linear intensity function</i>
------------------	--------------------------------------------------------------------------------------------------------------------------------------------------------

Description

Sample zero-truncated NHPPP times from a linear intensity function using the inversion method, optionally using an rstream generator

Usage

```
ztdraw_sc_linear(intercept, slope, t_min, t_max, atmost1 = FALSE)
```

Arguments

intercept	(double) the intercept
slope	(double) the slope
t_min	(double) the lower bound of the time interval
t_max	(double) the upper bound of the time interval
atmost1	(boolean) draw 1 event time

Value

a vector of at least 1 event times

Examples

```
x <- ztdraw_sc_linear(intercept = 0, slope = 0.2, t_min = 0, t_max = 10)
```

ztdraw_sc_loglinear	<i>Simulate from a zero-truncated non homogeneous Poisson Point Process (zt-NHPPP) from (t_min, t_max) with a log-linear intensity function</i>
---------------------	-------------------------------------------------------------------------------------------------------------------------------------------------

Description

Sample zt-NHPPP times from an log-linear intensity function

Usage

```
ztdraw_sc_loglinear(intercept, slope, t_min, t_max, atmost1 = FALSE)
```

Arguments

intercept	(double) the intercept in the exponent
slope	(double) the slope in the exponent
t_min	(double) the lower bound of the time interval
t_max	(double) the upper bound of the time interval
atmost1	boolean, 1 event time

Value

a vector of at least 1 event times

Examples

```
x <- ztdraw_sc_loglinear(intercept = 0, slope = 0.2, t_min = 0, t_max = 10)
```

ztppp	<i>Simulate a zero-truncated homogeneous Poisson Point Process over (t_min, t_max]</i>
-------	----------------------------------------------------------------------------------------

Description

Simulate a zero-truncated homogeneous Poisson Point Process over (t_min, t_max]

Usage

```
ztppp(rate, t_min, t_max, atmost1 = FALSE)
```

Arguments

rate	(scalar, double) constant instantaneous rate
t_min	(scalar, double) lower bound of the time interval
t_max	(scalar, double) upper bound of the time interval
atmost1	boolean, draw at most 1 event time

Value

a vector of event times of size size

Examples

```
x <- ztppp(t_min = 0, t_max = 10, rate = 0.001)
```

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